Seminar in PDEs and Applied Mathematics

Celebration of 100th Session

Online mode - Brazil

October, 7th - 10th, 2024

Book of Abstracts

Seminar in PDEs and Applied Mathematics - SPDEAM

Organizing committee

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Welcome

On behalf of the Organizing Committee, it is a pleasure to welcome all the participants to the Conference celebrating the 100th Session of Seminar in PDEs and Applied Mathematics, to be held on October from 7 to 10, 2024 at the ZOOM platform in the online mode.

This Conference "Seminar in PDEs and Applied Mathematics" - SPDEAM is a mathematical meeting celebrating the 100th Session of the webinar Seminar in PDEs and Applied Mathematics, aimed at bringing together researchers and students from several of the World, in particular from South, Central and North America, Europe and Africa. The goal is to present and share recent research results on PDEs in a pleasant atmosphere which, hopefully, will help to strengthening cooperation links.

This seminar started at August, 2020 in Brazil, during the COVID-19 pandemic. The founders of this seminar were Professors Juan Limaco and Max Souza from Federal University Fluminense in Brazil and Mauro Rincon from Federal University of Rio de Janeiro. Later, the organizing committee of the seminar has grown and expanded with institutions from Europe and Latin America, making the seminar stronger and more solid and giving the seminar a more international nature. Many internationally renowned speakers from all over the world have participated in the seminar.

The members of the organizing committee wish to express their gratitude to all our invited speaker for their valuable contribution in the realization of this event.

The Organizing Committee

SEMINAR IN PDE's AND APPLIED MATHEMATICS

Celebration of the 100th session

from
October 7th to 10th
2024

SPEAKERS:

Claudianor Alves, Brazil
Jone Apraiz, Spain
Nicola Bellomo, Italy
Piermarco Cannarsa, Italy
José A. Carrillo, UK
Jean-Michel Coron, France
Maicon Correa, Brazil
Manuel Del Pino, UK
Luz De Teresa, Mexico
Marcia Federson, Brazil
Enrique Fernández-Cara, Spain
Luis Farah, Brazil
Genni Fragnelli, Italy
Manuel González-Burgos, Spain
Alex Himonas, USA
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Tiago Roux Oliveira, Brazil
Boyan Sirakov, Brazil
Panagiotis Souganidis, USA
Emmanuel Trêlat, France
Roberto Triggiani, USA
Marius Tucsnak, France
Enrico Valdinoci, Australia
George Yin, USA
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Seminar in PDEs and Applied Mathematics - "Celebration of Session 100th", October 7th to 10th, 2024

TIME (UTC-3)	Monday 7	TIME (UTC-3)	Tuesday 8	TIME (UTC-3)	Wednesday 9	TIME (UTC-3)	Thursday 10
8:00-8:20	**Opening ceremony**						
8:20-9:00	**Enrique Zuazua** Chair: Enrique Fernández-Cara	9:10-9:50	**Cristina Pignotti** Chair: Anna Doubova	8:20-9:00	**Manuel del Pino** Chair: Maurício Sepúlveda	8:30-9:10	**Enrique Fernández-Cara** Chair: Juan Límaco
9:10-9:50	**Pierre-Louis Lions** Chair: Enrique Fernández-Cara	10:00-10:40	**Enrico Valdinoci** Chair: Duván Cardona	9:10-9:50	**Jean-Michel Coron** Chair: Ademir Pazoto	9:20-10:00	**Tiago Roux Oliveira** Chair: Juan Límaco
10:00-10:40	**Panagiotis Souganidis** Chair: Diego A. Souza	10:50-11:30	**Patrizia Pucci** Chair: Duván Cardona	10:00-10:40	**Lionel Rosier** Chair: Ademir Pazoto	10:10-10:50	**José Antonio Carrillo** Chair: Luz de Teresa
10:50-11:30	**Nicola Bellomo** Chair: Anna Doubova	11:40-12:20	**Maicon Correa** Chair: Maurício Sepúlveda	10:50-11:30	**Emmanuel Trélat** Chair: Diego A. Souza	11:00-11:40	**Genni Fragnelli** Chair: Luz de Teresa
11:40-12:20	**Luz de Teresa** Chair: Juan Límaco	12:30-13:10	**Marius Tucsnak** Chair: Anna Doubova	11:40-12:20	**Jaqueline Mesquita** Chair: Diego A. Souza	11:50-12:30	**Piermarco Cannarsa** Chair: Diego A. Souza
12:30-14:00	**Lunch**	13:20:14:00	**Lunch**	12:30-14:00	**Lunch**	12:40-14:00	**Lunch**
14:00-14:40	**Irena Lasiecka** Chair: Roberto Capistrano-Filho	14:00-14:40	**Carlos Kenig** Chair: Marcelo Cavalcanti	14:00-14:40	**Karl Kunisch** Chair: Felipe W. Chaves-Silva	14:00-14:40	**Alex Himonas** Chair: Roberto Capistrano-Filho
14:50-15:30	**Otared Kavian** Chair: Roberto Capistrano-Filho	14:50-15:30	**Manuel González-Burgos** Chair: Marcelo Cavalcanti	14:50-15:30	**Robert Lipton** Chair: Felipe W. Chaves-Silva	14:50-15:30	**Jone Apraiz** Chair: Roberto Capistrano-Filho
15:40-16:20	**Roberto Triggiani** Chair: Anna Doubova	15:40-16:20	**Felipe Linares** Chair: Diego A. Souza	15:40-16:20	**João Rodrigues** Chair: Sandra Malta	15:40-16:20	**Boyan Sirakov** Chair: Anna Doubova
16:30-17:10	**Wladimir Neves** Chair: Anna Doubova	16:30-17:10	**Luis Gustavo Farah** Chair: Diego A. Souza	16:30-17:10	**Marcia Federson** Chair: Sandra Malta	16:30-17:10	**George Yin** Chair: Anna Doubova
		17:20-18:00	**Claudianor Alves** Chair: Rui Almeida			17:20-17:40	**Closing ceremony**

	Rio de Janeiro, Brazil -03	Chicago, Illinois, USA CDT	London, UK BST	Madrid, Spain CEST	Perth, Australia AWST	Shanghai, China CST
	Mon Oct 7 **08:00**	Mon Oct 7 **06:00**	Mon Oct 7 **Noon**	Mon Oct 7 **13:00**	Mon Oct 7 **19:00**	Mon Oct 7 **19:00** Seventh Day of the National Day Golden Week
Best time	Mon Oct 7 **09:00**	Mon Oct 7 **07:00**	Mon Oct 7 **13:00**	Mon Oct 7 **14:00**	Mon Oct 7 **20:00**	Mon Oct 7 **20:00** Seventh Day of the National Day Golden Week
	Mon Oct 7 **10:00**	Mon Oct 7 **08:00**	Mon Oct 7 **14:00**	Mon Oct 7 **15:00**	Mon Oct 7 **21:00**	Mon Oct 7 **21:00** Seventh Day of the National Day Golden Week
Best time	Mon Oct 7 **11:00**	Mon Oct 7 **09:00**	Mon Oct 7 **15:00**	Mon Oct 7 **16:00**	Mon Oct 7 **22:00**	Mon Oct 7 **22:00** Seventh Day of the National Day Golden Week
	Mon Oct 7 **Noon**	Mon Oct 7 **10:00**	Mon Oct 7 **16:00**	Mon Oct 7 **17:00**	Mon Oct 7 **23:00**	Mon Oct 7 **23:00** Seventh Day of the National Day Golden Week
	Mon Oct 7 **13:00**	Mon Oct 7 **11:00**	Mon Oct 7 **17:00**	Mon Oct 7 **18:00**	Tue Oct 8 **Midnight**	Tue Oct 8 **Midnight**

Rio de Janeiro, Brazil -03	Chicago, Illinois, USA CDT	London, UK BST	Madrid, Spain CEST	Perth, Australia AWST	Shanghai, China CST
Mon Oct 7 **14:00**	Mon Oct 7 **Noon**	Mon Oct 7 **18:00**	Mon Oct 7 **19:00**	Tue Oct 8 **01:00**	Tue Oct 8 **01:00**
Mon Oct 7 **15:00**	Mon Oct 7 **13:00**	Mon Oct 7 **19:00**	Mon Oct 7 **20:00**	Tue Oct 8 **02:00**	Tue Oct 8 **02:00**
Mon Oct 7 **16:00**	Mon Oct 7 **14:00**	Mon Oct 7 **20:00**	Mon Oct 7 **21:00**	Tue Oct 8 **03:00**	Tue Oct 8 **03:00**
Mon Oct 7 **17:00**	Mon Oct 7 **15:00**	Mon Oct 7 **21:00**	Mon Oct 7 **22:00**	Tue Oct 8 **04:00**	Tue Oct 8 **04:00**
Mon Oct 7 **18:00**	Mon Oct 7 **16:00**	Mon Oct 7 **22:00**	Mon Oct 7 **23:00**	Tue Oct 8 **05:00**	Tue Oct 8 **05:00**

Summary

Plenary Talks

Seminar in PDEs and Applied Mathematics - Celebration of the 100th Session
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On existence of normalized solutions to some classes of elliptic problems with L^2-supercritical growth

Claudianor Alves[1]

In this talk, we will show some recent results involving the existence of normalized solutions to the following classes of elliptic problems in a bounded domain

$$\begin{cases} -\Delta u = \lambda u + f(u) \quad \text{in} \quad \Omega, \\ u(x) > 0 \quad \text{in} \quad \Omega, \\ u = 0 \quad \text{on} \quad \partial\Omega, \\ \displaystyle\int_\Omega |u|^2 dx = a^2, \end{cases} \tag{1}$$

and in the whole space $\mathbb{R}^N$ below

$$\begin{cases} -\Delta u + V(x)u = \lambda u + f(u) \;\text{ in }\; \mathbb{R}^N, \\ u(x) > 0 \quad \text{in} \quad \mathbb{R}^N, \\ \displaystyle\int_{\mathbb{R}^N} |u(x)|^2 dx = a^2. \end{cases} \tag{2}$$

In Problem (1), $\Omega \subset \mathbb{R}^N$ with $N \geq 2$ is a smooth bounded domain and f is a continuous function that fulfills the L^2-subcritical or L^2-supercritical growth. In particular, f is supposed to possess a critical exponential growth at infinity in the Trudinger-Moser sense when $N = 2$. In Problem (2), $V \in L^\infty(\mathbb{R}^N)$ is a positive $\mathbb{Z}^N$-periodic function and $f \in C(\mathbb{R}^N)$ with L^2-supercritical growth for $N \geq 3$ and critical exponential growth for $N = 2$. In both problems, $a > 0$ is small and $\lambda \in \mathbb{R}$ is an unknown parameter that appears as a Lagrange multiplier.

[1]Universidade Federal de Campina Grande, Brazil, email: coalvesbr@yahoo.com.br

Some geometric and fluid-solid interaction inverse problems in one dimension

Jone Apraiz[2], Anna Doubova, Enrique Fernández-Cara and Masahiro Yamamoto

Last decades, the analysis and solution of inverse problems has increased a lot because of their importance in many applications: elastography and medical imaging, seismology, potential theory, ion transport problems or chromatography and other similar fields. In this talk we will see some particular results we have obtained related to that area.

First, as an introduction to the work done by the research group, we will review briefly some geometric inverse problems we have studied for the heat, wave and Burgers equations (they can be read in [1] and [3]). Then, we will consider a one-dimensional fluid-solid interaction model governed by the Burgers equation with a time varying interface. This is a preliminary simplified version of other more complicate and more realistic models in higher dimensions. We will see the results we have obtained for the inverse problem of determining the shape of the interface from Dirichlet and Neumann data at one end point of the spatial interval. In particular, we will show uniqueness results and some conditional stability estimates. For the proofs, it will be seen that some lateral estimates have been used and adapted, which rely on appropriate Carleman and interpolation inequalities (following results in [6]).

The main results that will be shown in this talk have been published in [2]. Other related works are [4], about control results for simplified one-dimensional models of fluid-solid interaction, and [5], about large time behavior for a simplified n-dimensional model of the same type interaction

References

[1] J. Apraiz, J. Cheng, A. Doubova, E. Fernández-Cara, M. Yamamoto, *Uniqueness and numerical reconstruction for inverse problems dealing with interval size search*, Inverse Problems & Imaging **16** (3), (2022), 569–594.

[2] J. Apraiz, A. Doubova, E. Fernández-Cara, M. Yamamoto, *Inverse problems for one-dimensional fluid-solid interaction models*, Communications on Applied Mathematics and Computation, (2024).

[3] J. Apraiz, A. Doubova, E. Fernández-Cara, M. Yamamoto, *Some Inverse Problems for the Burgers Equation and Related Systems*, Communications in Nonlinear Science and Numerical Simulation 107:106113-106136, (2022).

[2]Department of Mathematics, University of the Basque Country, email: jone.apraiz@ehu.eus

[4] A. Doubova, E. Fernández-Cara, *Some control results for simplified one-dimensional models of fluid-solid interaction*, Mathematical Models & Methods in Applied Sciences **15**, (2005) no. 5, 783–824.

[5] A. Munnier, E. Zuazua, *Large time behavior for a simplified n-dimensional model of fluid-solid interaction*, Communications in Partial Differential Equations **30**, (2005) no. 3, 377–417.

[6] M. Yamamoto, *Carleman estimates for parabolic equations and applications*, Inverse Problems **25**, (2009), 123013.

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Behavioral Crowd Dynamics: Towards Artificial Intelligence for Living Systems

Nicola Bellomo[3] and Jie Liao

This Lecture focuses on the study of human crowds, where the dynamics is influenced by social interactions and collective learning. The modeling requires a multiscale vision and accounts for the quality and geometry of the venue where the dynamics occur. The study is developed within a mathematical theory suitable to account for the complexity features of living systems.

Indeed, the quest towards mathematical tools to describe behavioral crowd dynamics should not forget about the ambitious goal of developing a mathematical theory of living systems.

The presentation is in three parts.

1. On a mathematics for behavioral crowds: *Pedestrian are living beings. Therefore it is necessary referring the dynamics of crowds to a mathematical theory of living systems. The specific features of behavioral crowds should be identified to avoid the use of mathematical structures of the inert matter, see* [1].

2. Mathematical tools and simulations: *The technical problem is the derivation of mathematical models of crowd dynamics. Simulations should address the validation of models and support crisis managers in crisis situations, see* [2].

3. A forward took to an Artificial Intelligence platform: *Models and computational tools can be improved, analytic problems are challenging, but it is also useful understanding how Artificial Intelligence can contribute to safety, see* [3].

References

[1] Nicola Bellomo, Abdelghani Bellouquid, Livio Gibelli, and Nisrine Outada, **A Quest Towards a Mathematical Theory of Living Systems**, Birkhäuser-Springer, New York, (2017).

[2] Nicola Bellomo, J. Liao, A. Quaini, L. Russo, and C. Siettos, Human behavioral crowds review, critical analysis, and research perspectives, *Math. Models Methods Appl. Sci.*, **33**, 1611-1659, (2023).

[3] Nicola Bellomo, Marina Dolfin, and Jie Liao, Life and Self-Organization on the Way to Artificial Intelligence for Collective Dynamics, *Physics of Life Reviews*, **22**, 1–8, (2024).

[3]University of Granada, Spain, email: nicola.bellomo@ugr.es, http://www.modelingnature.org/nicola-bellomo

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Reconstruction of degeneracy region for a class of parabolic equations

Piermarco Cannarsa[4], Anna Doubova and Masahiro Yamamoto

We consider an inverse problem of reconstructing a degeneracy point in the diffusion coefficient of a one-dimensional parabolic equation by measuring the normal derivative on one side of the boundary. We study the sensitivity of the inverse problem to the initial data. We give sufficient conditions on the initial data for uniqueness and stability with a one-point measurement and provide examples of positive and negative results, supporting our theoretical analysis by numerical tests. We also present more general uniqueness results for the identification of both degeneracy and initial data by boundary measurements distributed over time. The method of proof is based on the representation of the solution by means of Bessel functions of the first type.

[4]Department of Mathematics, University of Rome Tor Vergata, email: cannarsa@axp.mat.uniroma2.it

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Nonlocal Aggregation-Diffusion Equations: fast diffusion and partial concentration

José A. Carrillo[5]

We will discuss several recent results for aggregation-diffusion equations related to partial concentration of the density of particles. Nonlinear diffusions with homogeneous kernels will be reviewed quickly in the case of degenerate diffusions to have a full picture of the problem. Most of the talk will be devoted to discuss the less explored case of fast diffusion with homogeneous kernels with positive powers. We will first concentrate in the case of stationary solutions by looking at minimisers of the associated free energy showing that the minimiser must consist of a regular smooth solution with singularity at the origin plus possibly a partial concentration of the mass at the origin. We will give necessary conditions for this partial mass concentration to and not to happen. We will then look at the related evolution problem and show that for a given confinement potential this concentration happens in infinite time under certain conditions. We will briefly discuss the latest developments when we introduce the aggregation term. This talk is based on a series of works in collaboration with M. Delgadino, J. Dolbeault, A. Fernández, R. Frank, D. Gómez-Castro, F. Hoffmann, M. Lewin, and J. L, Vázquez.

[5]Mathematical Institute, University of Oxford, email: carrillo@maths.ox.ac.uk

Boundary stabilization of 1-D hyperbolic systems

Jean-Michel Coron[6]

Hyperbolic systems in one-space dimension appear in various real-life applications (navigable rivers and irrigation channels, heat exchangers, chemical reactors, gas pipes, road traffic, chromatography, ...). This presentation will focus on the stabilization of these systems by means of boundary controls. Stabilizing feedback laws will be constructed. It will include explicit feedback laws that have been implemented for the regulation of the rivers La Sambre and La Meuse, integral action, stabilization in the optimal time, the backstepping approach, output stabilization, and robustness issues.

[6]Sorbonne University, France, email: jean.m.coron@gmail.com

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Numerical Schemes for Elliptic, Parabolic and Hyperbolic PDEs in Porous Media Flow

Maicon R. Correa[7], Marcio A. Murad

In this talk, we briefly discuss a mathematical model for multiphase flow in Poroelastic media, where the conservation of the corresponding component densities replaces the traditional black-oil conservation volume equations at standard conditions [1, 2]. Then, we present a sequential iterative algorithm for numerically solving the proposed model, which is composed of a nonlinear system of partial differential equations and discuss the use of different numerical methods for the (linearized) parabolic and elliptic subproblems of flow and geomechanics. We also present an innovative semi-discrete central-upwind finite volume scheme for the system of hyperbolic conservation laws capable of capturing spatial and temporal variability in porosity. This scheme obviates the necessity of adopting operation splitting schemes for the storativity in the transport equations, as performed in previous works [3]. The continuous-in-time semi-discrete formulation is then integrated using Strong Stability Preserving Runge-Kutta (SSPRK) schemes. The potential of the proposed scheme is illustrated in numerical simulations of black-oil flow problems in poroelastic media.

References

[1] John A. Trangenstein and John B. Bell. Mathematical structure of the black-oil model for petroleum reservoir simulation. *SIAM Journal on Applied Mathematics*, 49(3):749–783, 1989.

[2] Maicon R. Correa and Marcio A. Murad. A fixed-stress-split scheme for a black-oil model inporoelastic media. 2024. Submitted.

[3] Maicon R. Correa and Marcio A. Murad. A new sequential method for three-phase immiscible flow in poroelastic media. *Journal of Computational Physics*, 373:493 – 532, 2018.

[7]Department of Applied Mathematics, University of Campinas, email: maicon@ime.unicamp.br

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Overhanging solitary waves in the Water Wave Problem

Manuel del Pino [8]

In the classical Water Wave Problem, we construct new stationary overhanging solitary waves by a procedure resembling desingularization of the gluing of constant mean curvature surfaces by tiny catenoid necks. The solutions here predicted have long been numerically detected. This is joint work with Juan Davila, Monica Musso, and Miles Wheeler. The waves are solitary with constant vorticity, and exist when an appropriate dimensionless gravitational constant $g > 0$ is sufficiently small. Our construction in [1] involves combining three explicit solutions to related problems: a disk of fluid in rigid rotation, a linear shear flow in a strip, and a rescaled version of an exceptional domain discovered by Hauswirth, Hélein, and Pacard [2]. The method developed here is related to the construction of constant mean curvature surfaces through gluing.

References

[1] J. Dávila, M. del Pino, M. Musso, M. H. Wheeler *Overhanging solitary water waves.* 93 pages. Preprint arXiv:2409.01182v1

[2] L. Hauswirth, F. Helein, and F. Pacard , *On an overdetermined elliptic problem.* Pacific J. Math., 250 (2011), pp. 319–334.

[8] Department of Mathematical Sciences, University of Bath, email: mdp59@bath.ac.uk

Changes of roles in Stackelberg control strategies

Luz de Teresa [9], Bianca Calsavara, Enrique Fernández-Cara and José Antonio Villa-Morales

In this talk we will present a Stackelberg strategy to control parabolic equations. We will act on the system with two controls: one will act as a leader and the other as a follower. We will discuss the different problems as main objective a null controllability one and the follower been an optimization one. Then, we will present a result inverting the objectives of the leader and the follower. The leader will have an optimization target and the follower a null controllability one, [1] We briefly discuss a similar problem but for a wave equation [2].

References

[1] B. Calsavara, E. Fernández-Cara, L. de Teresa, and J.-A. Villa, New results concerning the hierarchical control of linear and semilinear parabolic equations. ESAIM Control Optim. Calc. Var. 28 (2022), Paper No. 14, 26 pp.

[2] L. de Teresa, and J.-A. Villa. A new hierarchical control for the wave equation. Submitted

[9] Instituto de Matemáticas, UNAM, email:ldeteresa@im.unam.mx

Stability for Generalized Stochastic Equations

Fernanda Andrade da Silva, Everaldo de Mello Bonotto and Márcia Federson[10]

Generalized ordinary differential equations (generalized ODEs) are based on the non-absolute integration theory due to Jaroslav Kurzweil and Ralph Henstock and are known to cover many types of equations, as for instance, measure functional differential equations, dynamic equations on time scales and integral equations (see e.g., [1] and the references therein). By considering belated partial divisions in the classic Kurzweil integral, the authors of [3] introduced a new integral which contains the Itô-Henstock integral for functions taking values in spaces of Hilbert-Schmidt operators. Furthermore, they defined a new class of equations, called generalized stochastic equations (GSEs), in such a way that classic stochastic differential equations fall into special cases of GSEs. In this presentation, based in [2], we provide stability theory for GSEs by means of Lyapunov functionals and we discuss the main results.

References

[1] Bonotto, E.M.; Federson, M.; Mesquita, J. *Generalized Ordinary Differential Equations in Abstract Spaces and Applications.* Hoboken NJ, First Edition, John Wiley & Sons, 2021.

[2] Andrade da Silva, F.; Bonotto, E.M.; Federson, M. *Stability for generalized stochastic equations.* Stoch. Proc. Appl., **173**, (2024), 104358.

[3] Bonotto, E.M.; Collegari, R.; Federson, M.; Gill, T. *Operator-valued stochastic differential equations in the context of Kurzweil-like equations.* J. Math. Anal. Appl. **527**, (2023), 127464.

[10] Department of Mathematics, University of São Paulo, São Carlos, email: federson@icmc.usp.br

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On some inverse problems concerning the domain and/or the coefficients of a PDE

Enrique Fernández-Cara[11]

In this contribution, I will consider several inverse problems for PDEs. The aim is to identify either the spatial domain or the coefficients of an evolution PDE from boundary observations produced by the solution. As usual, there are three main questions to be interested in: uniqueness, stability and reconstruction. Several results will be mentioned with detail. They have been obtained in collaboration with several people: J. Apraiz, A. Doubova, F. Maestre, M. Yamamoto and others. I will also recall several important applications where the results become useful.

References

[1] Apraiz, J.; Doubova, A.; Fernández-Cara, E.; Yamamoto, M. Some inverse problems for the Burgers equation and related systems. Commun. Nonlinear Sci. Numer. Simul. 107 (2022), Paper No. 106113, 23 pp.

[2] Carvalho, P.P.; Doubova, A.; Fernández-Cara, E.; Rocha de Faria, J. Some new results for geometric inverse problems with the method of fundamental solutions. Inverse Probl. Sci. Eng. 29 (2021), no. 1, 131–152.

[3] Doubova, A.; Fernández-Cara, E.; González-Burgos, M.; Ortega, J.H. A geometric inverse problem for the Boussinesq system. Discrete Contin. Dyn. Syst. Ser. B 6 (2006), no. 6, 1213–1238.

[4] Fernández-Cara, E.; Maestre, F. An inverse problem in elastography involving Lamé systems. J. Inverse Ill-Posed Probl. 26 (2018), no. 5, 589–605.

[11] Department EDAN and IMUS, University of Sevilla (Spain), email: cara@us.es

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On the mass-critical inhomogeneous NLS equation

Luiz Gustavo Farah[12]

We consider the inhomogeneous nonlinear Schrödinger (INLS) equation

$$iu_t + \Delta u + |x|^{-b}|u|^{\frac{4-2b}{N}}u = 0, \ \ x \in \mathbb{R}^N,$$

with $N \geq 1$ and $0 < b < 1$, which is a generalization of the classical nonlinear Schrödinger equation (NLS). Since the scaling invariant Sobolev index is zero, the equation is called mass-critical.

In this talk we discuss some blow-up results in the non-radial setting, obtained in collaboration with Mykael Cardoso (UFPI-Brazil).

This work is partially supported by CNPq, CAPES and FAPEMIG-Brazil.

[12] Department of Mathematics, Universidade Federal de Minas Gerais (UFMG), email: farah@mat.ufmg.br

Some new results on the beam equation

Genni Fragnelli[13]

We will discuss a very recent approach to the study of beam-like equations. After introducing the needed mathematical setting for these classes of problems, we will provide some existence results and the description of the behaviour of the solutions for some concrete models.

[13]University of Tuscia, email: genni.fragnelli@unitus.it

Controllability of parabolic problems with distributed controls supported on a hyperplane

Farid Ammar-Khodja, Assia Benabdallah, Manuel González-Burgos[14],
Morgan Morancey and Luz de Teresa

In this talk we will present some new results on controllability of N-dimensional parabolic problems at time $T > 0$ in cylindrical domains $\Omega = \Omega_1 \times (0, \pi) \subset \mathbb{R}^N$ with distributed controls supported on $\omega \times \{x_0\}$, with $\omega \subset \Omega_1$, a nonempty open subset, and $x_0 \in (0, \pi)$. If the evolution operator can be tensorized, we will prove the existence of a minimal control time which depends on x_0. The main ingredient that we will use is the moment method in a N-dimensional framework and the so-called Lebeau-Robbiano spectral inequality for the eigenvectors of the transverse operator.

References

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[14]Dpto. Ecuaciones Diferenciales y Análisis Numérico and, Instituto de Matemáticas de la Universidad de Sevilla, Universidad de Sevilla, Spain, email: manoloburgos@us.es

Initial-boundary value problems for nonlinear Schrödinger equations

Alex Himonas[15]

In this talk we shall discuss the progress made during the last several years in the study of well-posedness of initial-boundary value problems for nonlinear dispersive equations via the Fokas method. Our focus is on nonlinear Schrödinger equations. Using the Fokas solution formula for the forced linear problem we derive good linear estimates for data in Sobolev spaces and forcing in appropriate Bourgain spaces, analogues to those for initial value problems. Then, using these linear estimates and the multilinear estimates indicated by the forcing we show that the iteration map defined by the Fokas solution formula is a contraction in appropriate solution spaces. While the Fokas solution formula makes our approach analogous to that used for initial value problems, the presence of boundary introduces more interesting situations. The talk is based on work with A. Fokas, D. Mantzavinos and F. Yan.

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[15] Department of Mathematics, University of Notre Dame, email:himonas.1@nd.edu

Polynomial stability of an abstract system with local damping

Otared Kavian[16] and Qiong Zhang

We consider a class of wave equations of the type

$$\partial_{tt}u + Lu + B\partial_t u = 0$$

with a self-adjoint operator L, and various types of damping represented by the term $B\partial_t u$. By establishing appropriate and raher precise estimates on the resolvent of an associated operator A on the imaginary axis of $\mathbb{C}$, we prove polynomial decay of the semigroup $\exp(-tA)$ generated by that operator. We point out that the rate of decay depends strongly on the concentration of eigenvalues and that of the eigenfunctions of the operator L. We give several examples of application of our abstract result, showing in particular that when $\Omega \subset \mathbb{R}^N$, and

$$L = -\Delta \quad \text{with} \quad D(L) := \left\{ u \in H_0^1(\Omega) \; ; \; \Delta u \in L^2(\Omega) \right\},$$

and $B = (-\Delta)^\theta$ with $0 \leq \theta \leq 1$, for instance when $N = 2$, and the domain $\Omega := (0, L_1) \times (0, L_2)$ the decay rate of the energy is different depending on whether the ratio L_1^2/L_2^2 is rational, or irrational but algebraic. An analogous result holds when $N \geq 3$.

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[16] Laboratoire de Mathématiques Appliquées, UMR 8100 du CNRS, Université Paris–Saclay (site UVSQ), 45 avenue des Etats Unis, 78035 Versailles Cedex, France, email: otared.kavian@uvsq.fr

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Schrodinger evolutions for variable coefficient non-local operators

Carlos Kenig[17]

I will discuss work in progress with D.Pilod, G.Ponce and L. Vega on the well-posedness of semilinear Schrodinger evolutions associated with nonlocal variable coefficient operators. I will also discuss unique continuation results for such operators. These results depend on the spectral analysis of the underlying variable coefficient selfadjoint variable coefficient second order elliptic operator, which we develop in detail.

[17]Department of Mathematics University of Chicago, USA, email: ckenig@uchicago.edu

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Solving the Hamilton Jacobi Bellman equation of optimal control: towards taming the curse of dimensionality

Behzad Azmi, S. Dolgov, Karl Kunisch[18], Donato Vaquez-Varas and Daniel Walter

The characterization of optimal feedback for nonlinear dynamical systems involves solving a Hamilton-Jacobi-Bellman equation. This is a nonlinear first order hyperbolic equation in a space whose dimension is that of the state-space of the underlying system. Thus solving this system in practice one is confronted with a curse of dimensionality.

In this talk we present techniques which either, in part, circumvent the necessity of solving the HJB equations directly, or use system reduction techniques to alleviate the difficulty associated with high dimensions. Specifically, we briefly describe experience with a succinct use of policy iteration, we introduce a data-driven technique which exploits higher order information, and finally we explain the 'Averaged Feedback Learning Scheme'.

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[18] Department of Mathematics and Scientific Computing, University of Graz, and Radon Institute, Austrian Academy of Sciences. email: karl.kunisch@uni-graz.at

Control of large sustained oscillations in flow-structure interactions

Irena Lasiecka[19]

Flow-structure interactions are ubiquitous in nature and in everyday life. Flow or fluid by interacting with structural elements can lead to oscillations, hence impacting stability or even safety. Thus problems such as attenuation of turbulence or flutter in an oscillating structure [Tacoma bridge], flutter in tall buildings, fluid flows in flexible pipes, in nuclear engineering flows about fuel elements and heat exchanger vanes are just few prime examples of relevant applications which place themselves at the frontier of interests in applied mathematics. In this lecture we shall describe mathematical models describing the phenomena, These are represented by a 3 D Euler Equation coupled to a **nonlinear** dynamic elasticity on a 2 D manifold. Strong boundary-type coupling at the interface between the two media is at the center of the analysis. This provides for a rich mathematical structure, opening the door to several unresolved problems in the area of nonlinear PDE's, dynamical systems, related harmonic analysis and differential geometry. This talk aims at providing a brief overview of recent developments in the area along with a presentation of some new methodology addressing the issues of asymptotic control to coherent structure and stability of the relevant dynamics.

Part of this work performed while the author was a member of the MSRI program "Mathematical problem in fluid dynamics"at the University of California Berkeley during the Spring 2021 and Fall 2023 semester (NSF DMS -1928930).

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[19]Department of Mathematical Sciences, the University of Memphis and Department of Mathematics University of Virginia. lasiecka@memphis.edu

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On special properties of solutions to Camassa-Holm equation and related models

Felipe Linares[20]

We will discuss unique continuation properties for solutions to the b-family of equations. This includes the Camassa-Holm and the Degasperi-Procesi models. We prove that for both, the initial value problem and the periodic boundary value problem, the unique continuation results found in [1] are optimal. More precisely, the result established there for the constant $c_0 = 0$ fails for any constant $c_0 \neq 0$.

This is a joint work with Christian Hong and Gustavo Ponce from UC, Santa Barbara.

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[20] IMPA, Rio de Janeiro, Brazil, email: linares@impa.br

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Mean Field Games: Well-posedness, singularities and beyond (?)

Pierre-Louis Lions[21]

We present in this talk a general theory for the well-posedness of (extended) Mean Field Games models.

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[21] Collège de France, email: pierre-louis.lions@college-de-france.fr

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Fracture as Emergent Phenomena

Robert P. Lipton[22] and Debdeep Bhattacharya

A nonlocal model for dynamic brittle damage is introduced consisting of two phases, one elastic and the other inelastic. Evolution from the elastic to the inelastic phase depends on material strength. Existence and uniqueness of the displacement-failure set pair follow fro m an initial value problem describing the evolution. The displacement-failure pair satisfies energy balance. The length of nonlocality ϵ is taken to be small relative to the domain in $\mathbb{R}^d$, $d = 2, 3$. The strain is formulated as a difference quotient and the strain evolution is on a subset of $\mathbb{R}^d \times \mathbb{R}^d$. Projection of this evolution onto $\mathbb{R}^d$ provides an energy balance between external energy, elastic energy and damage energy including fracture energy. For any prescribed loading the deformation energy resulting in material failure over a region R is uniformly bounded as $\epsilon \to 0$. For fixed ϵ, the failure energy is nonzero for $d-1$ dimensional regions R associated with flat crack surfaces. Calculation shows this failure energy is the Griffith fracture energy given by the energy release rate multiplied by area for $d = 3$ (or length for $d = 2$). The field theory is shown to recover a solution of Naiver's equation outside a propagating flat traction free crack in the limit of vanishing spatial nonlocality. The theory and simulations presented here corroborate recent experimental findings [1] that cracks follow the location of maximum energy dissipation inside the intact material. Simulations illustrate fracture evolution through generation of a traction free internal boundary as a wake left behind a moving strain concentration.

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[22] Department of Mathematics, Louisiana State University, email: lipton@lsu.edu

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A new concept of periodicity

Jaqueline Godoy Mesquita[23]

In this talk, we will introduce a more general concept of periodicity and present some applications.

[23]Department of Mathematics, Louisiana State University, email: lipton@lsu.edu

An Eulerian-Lagrangian formulation for compressible Euler Equations

Wladimir Neves[24] and Christian Olivera

In this talk, we present an Eulerian-Lagrangian formulation for the compressible isentropic Euler equations with a general pressure law. Relying on the Lagrangian formulation, we show a short-time solution which is unique in the framework of Bessel space $H_p^\beta(\mathbb{T}^d)$ for $\beta > \frac{d}{p} + 1$. The proof is precise, relatively simple, and the result can be stated economically. To our knowledge, this is the first time the Lagrangian formulation has been employed to show local wellposedness for compressible Euler equations. Moreover, it can be adaptable to $C^{k,\alpha}$ spaces, Sobolev spaces, and notably, Besov spaces.

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[24]Instituto de Matemática, Universidade Federal do Rio de Janeiro, Brazil. E-mail: *wladimir@im.ufrj.br*

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Exponential stability for semilinear viscoelastic wave equations with delay feedback

Elisa Continelli and Cristina Pignotti[25]

We consider a class of semilinear second-order evolution equations with viscoelastic damping and time-varying delay feedback. The memory kernel is exponentially decaying, in order to have exponential stability for the model without the nonlinearity and the delay feedback. The time delay function is assumed to be continuous and bounded. We are interested in studying the model's well-posedness and the asymptotic behavior of the solutions. Under a suitable assumption involving the delay feedback and the model's parameters, we can prove that solutions corresponding to small initial data are globally defined and satisfy an exponential decay estimate.

[25]Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica, University of L'Aquila, email: cristina.pignotti@univaq.it

Double critical elliptic equations in $\mathbb{R}^N$

Patrizia Pucci[26]

The talk presents some recent results on existence of solutions for double critical equations in $\mathbb{R}^N$. The first part deals with the existence of nontrivial nonnegative solutions of model parametric elliptic equations in $\mathbb{R}^N$ involving a possibly supercritical term in Sobolev sense, and a nonlocal term with the upper Hardy-Littlewood-Sobolev critical exponent. Under some conditions, we describe the precise parametric range of existence and nonexistence of a nonnegative solution. Furthermore, in a slightly smaller range, a second nontrivial nonnegative solution is constructed. Additionally, infinitely many solutions, with energy asymptotic behavior, are also obtained if the growth near the origin is concave. These results, which are inspired by the pioneering work of Alama and Tarantello [1] in the local case of Dirichlet problems in bounded domains, are obtained by combining variational methods, Leray-Schauder degree theory, and the Krasnoselskii genus via biorthogonal functionals in separable and reflexive Banach spaces.

The last part of the talk shows other existence results for different critical equations in $\mathbb{R}^N$, obtained with different proof techniques. Finally, some open problems are also given.

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[26] Dipartimento di Matematica e Informatica, Università degli Studi di Perugia, email: patrizia.pucci@unipg.it

Kirchhoff's formula and numerical control of the wave equation

Antoine Benoit, Romain Loyer and Lionel Rosier[27]

A new Kirchhoff's formula involving wavefronts is established for the solution to the wave equation with homogeneous Dirichlet boundary conditions on a strictly convex bounded open set in $\mathbb{R}^3$. Under some assumptions about the rays of geometrical optics, one obtains solutions of the wave equation on strictly convex bounded open sets vanishing at some positive time on the support of their initial conditions, as it occurs in $\mathbb{R}^3$ with Huygens' principle. That property is used to design a direct method for the numerical control of the wave equation in strictly convex bounded open sets in $\mathbb{R}^3$.

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[27] LMPA, Université du Littoral Côte d'Opale (Calais, France). email: Lionel.Rosier@univ-littoral.fr

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Extremum and Nash Equilibrium Seeking through Delays and PDEs

Tiago Roux Oliveira[28]

The development of extremum seeking (ES) has progressed, over the past hundred years, from static maps, to finite-dimensional dynamic systems, to networks of static and dynamic agents. Extensions from ODE dynamics to maps and agents that incorporate delays or even partial differential equations (PDEs) is the next natural step in that progression through ascending research challenges. This lecture reviews results on the algorithm design and theory of ES for such infinite-dimensional systems. Both gradient and Newton algorithms are presented, for both hyperbolic and parabolic dynamics: delay equations, wave equations, and reaction-advection-diffusion equations. Methods are introduced for single-agent optimization and extended to non-cooperative game scenarios of the model-free kind. Even heterogeneous PDE games, such as a duopoly with one parabolic and one hyperbolic agent, are considered. Several engineering applications are touched on for illustration, including neuromuscular electrical stimulation, additive manufacturing modeled by the Stefan PDE, biological reactors, deep-sea cable-actuated source seeking, oil-drilling systems, and flow-traffic control for urban mobility.

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[28]Department of Electronics and Telecommunication Engineering, State University of Rio de Janeiro (UERJ), email: tiagoroux@uerj.br

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The Method of the Nehari manifold on cones

Denilson S. Pereira, João R. Santos Jr.[29] and Felipe F. Silva

In[2], among other things, A. Szulkin and T. Weth study a series of elliptic partial differential problems involving continuous (and possibly non-differentiable) nonlinearities whose primitives satisfy the so called superquadratic condition at infinity. In order to apply the Nehari method to find ground-state and other kind of solutions, the authors introduce an interesting approach to overcome the lack of a C^1 structure for the Nehari set. In fact, under suitable conditions on the functional $I : E \to \mathbb{R}$ and on the Banach space E, is proved the existence of a homeomorphism between the Nehari set $\mathcal{N}$ associated to I and the unit sphere $\mathcal{S}$ of E. Such a homeomorphism allows them to define an auxiliary functional $\Psi : \mathcal{S} \to \mathbb{R}$ with the convenient property that the existence of critical points for Ψ implies in the existence of critical points of I. Due to conditions imposed on the Banach space E, the unit sphere $\mathcal{S}$ is a C^1 manifold and, for this reason, the task of looking for critical points of Ψ is a more treatable problem.

Still in [2], it is revisited a classical paper of Benci and Cerami (see [1]) which relate the number of solutions of a certain elliptic PDE with the topology of the domain where the equation is considered. The authors are able to improve the main results in [1] by assuming weaker conditions on the nonlinearity. However, since the idea is to obtain positive solutions, the Nehari set in this case is no longer homeomorphic to the hole unit sphere $\mathcal{S}$, but to an open subset $\mathcal{S}^+$ of $\mathcal{S}$. As observed in [2], this new situation brings together some technical difficulties, as for instance the fact that $\mathcal{S}^+$ has a nonempty boundary in $\mathcal{S}$ and, therefore, the behaviour of minimizing sequences need to be carefully controlled near the boundary.

In this work, we generalize the ideas in [2] to find positive solutions of the Benci-Cerami problem for an abstract situation where the same sort of technical difficulty is occurring. In the general context, we establish an abstract method that allows us to study the existence of solutions for some classes of elliptic problems with continuous nonlinearities. The referred method is used to find ground state solutions to a sort of elliptic problems under different mathematical contexts.

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[29] Faculty of Mathematics, UFPA, email: joaojunior@ufpa.br

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Elliptic regularity estimates with optimized constants and applications

Boyan Sirakov[30] and Philippe Souplet

We revisit the classical theory of linear second-order uniformly elliptic equations in divergence form whose solutions have Hölder continuous gradients, and prove versions of the generalized maximum principle, the $C^{1,\alpha}$-estimate, the Hopf-Oleinik lemma, the boundary weak Harnack inequality and the differential Harnack inequality, in which the constant is optimized with respect to the norms of the coefficients of the operator and the size of the domain. Our estimates are complemented by counterexamples which show their optimality. We also give applications to the Landis conjecture and spectral estimates.

[30]Departamento de MatemÃ¡tica, PUC-Rio, email:bsirakov@puc-rio.br

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Optimal control of the Dyson Equation and large deviations for Hermitian random matrices

Charles Bertucci, Pierre-Louis Lions and Panagiotis Souganidis[31]

The control of the Dyson equation, which is the mean field equation for the eigenvalues of large random Hermitian matrices, leads to the study of a nonlinear Hamilton-Jacobi-type equation with singularities in the set of probability spaces. In the lecture, I will describe how to establish the well-posedness of the solutions of these equations. Then, I will use these results to obtain large deviations for the eigenvalues of large random Hermitian matrices.

[31] University of Chicago, email: souganidis@math.uchicago.edu

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Approximate control of parabolic equations with on-off shape controls by Fenchel duality

Emmanuel Trélat[32]

We consider the internal control of linear parabolic equations through *on-off shape* controls, i.e., controls of the form $M(t)\chi_{\omega(t)}$ with $M(t) \geq 0$ and $\omega(t)$ with a prescribed maximal measure.

We establish small-time approximate controllability towards all possible final states allowed by the comparison principle with nonnegative controls. We manage to build controls with constant amplitude $M(t) \equiv \overline{M}$. In contrast, if the moving control set $\omega(t)$ is confined to evolve in some region of the whole domain, we prove that approximate controllability fails to hold for small times.

The method of proof is constructive. Using Fenchel-Rockafellar duality and the bathtub principle, the on-off shape control is obtained as the bang-bang solution of an optimal control problem, which we design by relaxing the constraints.

Our optimal control approach is outlined in a rather general form for linear constrained control problems, paving the way for generalisations and applications to other PDEs and constraints.

This is a work with Camille Pouchol and Christophe Zhang (see [1]).

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[32] Laboratoire Jacques-Louis Lions, Sorbonne Université, email: emmanuel.trelat@sorbonne-universite.fr

Uniform Stabilization in Besov spaces of the Magnetohydrodynamics system by finite-dimensional interior localized static feedback controllers

Roberto Triggiani[33]

We consider the d-dimensional MagnetoHydroDynamics (MHD) system defined on a sufficiently smooth bounded domain, $d = 2, 3$ with homogeneous boundary conditions, and subject to external sources assumed to cause instability. The initial conditions for both fluid and magnetic equations are taken of low regularity. We then seek to uniformly stabilize such MHD system in the vicinity of an unstable equilibrium pair, in the critical setting of correspondingly low regularity spaces, by means of explicitly constructed, static, finite dimensional feedback controls, which are localized on an arbitrarily small interior subdomain. In additional, they will be minimal in number and explicitly constructed. The resulting space of well-posedness and stabilization is a suitable product space of tight Besov spaces for the fluid velocity component and the magnetic field component (each "close"to $L^3(\Omega)$ for $d = 3$). This is part of the ongoing work on uniform stabilization of fluids jointly with Irena Lasiecka and Buddhika Priyasad.

[33]University of Memphis and University of Virginia, USA, email: rtrggani@memphis.edu

PDE systems describing the motion of solids in a viscous fluid: wellposedness, control and long-time behaviour

Marius Tucsnak[34]

The study of systems describing the motion of rigid bodies in a fluid goes back to Euler and Kirchhoff, who considered the case of an ideal fluid subject undergoing a potential flow. The case of a single solid moving in a viscous incompressible fluid filling the whole space was considered much later, around 1980 by Weinberger and Serre. Well-posedness issues for the case when the solid-fluid system is contained in a bounded container or/and of several moving bodies has been intensively studied since 2000. In this talk we first recall some of these wellposedness results . We next describe some associated control problems, aimed to provide a new approach to the understanding of the swimming of aquatic organisms. We finally describe some recent results concerning the long-time behaviour of solutions.

[34]Institut de Mathématiques de Bordeaux (IMB), France, email: marius.tucsnak@u-bordeaux.fr

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Sheet happens (but only as the root of 1-s)

Enrico Valdinoci[35]

We discuss the regularity properties of two-dimensional stable s-minimal surfaces, presenting a robust estimate and an optimal sheet separation bound, according to which the distance between different connected components of the surface must be at least the square root of 1-s.

[35] University of Western Australia, email: enrico.valdinoci@uwa.edu.au

Computational Nonlinear Filtering: A Deep Learning Approach

Hongjiang Qian, George Yin[36] and Qing Zhang

Nonlinear filtering is a fundamental problem in signal processing, information theory, communication, control and optimization, and systems theory. In the 1960s, celebrated results on nonlinear filtering were obtained. Nevertheless, the computational issues for nonlinear filtering remained to be a long-standing and challenging problem. In this talk, in lieu of treating the stochastic partial differential equations for obtaining the conditional distribution or conditional measure, we construct finite-dimensional approximations using deep neural networks for the optimal weights. Two recursions are used in the algorithm. One of them is the approximation of the optimal weight and the other is for approximating the optimal learning rate.

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[36] Department of Mathematics, University of Connecticut, email: gyin@uconn.edu

Control and Machine Learning

Enrique Zuazua[37]

Systems control, or cybernetics, a term first coined by Ampère and later popularized by Norbert Wiener, refers to the science of control and communication in animals and machines. The pursuit of this field dates back to antiquity, driven by the desire to create machines that autonomously perform human tasks, thereby enhancing freedom and efficiency.

The objectives of control systems closely parallel those of modern Artificial Intelligence (AI), illustrating both the profound unity within Mathematics and its extraordinary capacity to describe natural phenomena and drive technological innovation.

In this lecture, we will explore the connections between these mathematical disciplines and their broader implications. We will also discuss our recent work addressing two fundamental questions: Why does Machine Learning perform so effectively? And how can data-driven insights be integrated into the classical applied mathematics framework, particularly in the context of Partial Differential Equations (PDE) and numerical methods?

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[37]Friedrich Alexander Universität Erlangen Nürnberg - Alexander von Humboldt Professorship, Germany. email: enrique.zuazua@fau.de

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